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NOTE

An Example of Precise Interpolation with a Spline Function

The rigorous mathematical formulas derived from potential theory for the force and potential in the field of a nonhomogeneous spheroid [1] are rather cumbersome for frequent use. In this note a description is given of a spline function of degree three that has been fitted to a series of ten points representing a segment of a theoretical rotation curve of the galactic system [2], [3]. The points to be fitted are given in Table 1; y is the circular velocity in the galactic plane in km/sec at a distance of x kiloparsec (1 parsec = 3.26 light years) from the galactic center. Clearly the points cannot easily be represented by some standard interpolation formula such as a polynomial.

TABLE 1

SEGMENT OF A GALACTIC ROTATION CURVE

x	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
y	244.0	221.0	208.0	208.0	211.5	216.0	219.0	221.0	221.5	220.0

The first method described in [2] (for unequally spaced arguments, in general) has been programmed for our IBM 7040 computer and the result, correct to four decimals, is

$$\begin{aligned} y(x) = 244.0 &- 24.8065(x-1) + 1.8065(x-1)_+^3 \\ &- 0.8391(x-2)_+^3 - 3.6437(x-3)_+^3 \\ &+ 2.9140(x-4)_+^3 - 1.0122(x-5)_+^3 \\ &+ 1.1349(x-6)_+^3 - 0.5272(x-7)_+^3 \\ &- 0.0261(x-8)_+^3 + 0.6315(x-9)_+^3 \\ &- 0.4386(x-10)_+^3, \end{aligned}$$

where the "plus" subscript after a bracket, e.g. $(x-c)_+^3$, indicates that the bracket has its algebraic value for $x \geq c$, and is zero for $x < c$. ("c" is here assumed positive).

A check of the errors produced by $y(x)$ for a series of intermediate x-values was made by comparing the predicted values with exact values from the theoretical rotation curve [3]. The errors were not greater than 1 part in 500. The author will be happy to send copies of the program to interested persons. I am grateful to the reviewer for pointing out some numerical errors in the original manuscript.

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